Reflection of electromagnetic waves from isotropic optically active media

1991 J. Phys.: Condens. Matter 3 529

(http://iopscience.iop.org/0953-8984/3/5/003)

J. Phys.: Condens. Matter 2 (1991) 529–538. Printed in the UK

Reflection of electromagnetic waves from isotropic optically active media

A I Miteva† and I J Lalov‡

† Department of Physics, Institute for Foreign Students, 1111 Sofia, Bulgaria

‡ Faculty of Physics, Sofia University, 1126 Sofia, Bulgaria

Received 13 March 1990, in final form 24 September 1990

Abstract. The paper investigates the reflection of linearly and circularly polarized electromagnetic waves both at the boundary between an isotropic non-active medium and an isotropic optically active medium and at the boundary between an optically active medium and a non-active medium. The components of the electric field and the intensity of the reflected and refracted waves are determined. The following differential reflection methods for studying the optical activity are proposed: usual reflection of left and right circularly polarized waves (reflection optical activity); total reflection of circularly polarized waves; total reflection of linearly polarized waves.

1. Introduction

Well known manifestations of the optical activity (i.e. gyrotropy) are the rotation of the polarization plane and the circular dichroism which are observed when electromagnetic waves (EMWs) propagate in an optically active medium. The normal EMWs (solutions of the Maxwell equations) in an isotropic gyrotropic medium are left circularly polarized (LCP) and right circularly polarized (RCP) waves of different refractive indices and of different absorption coefficients. The gyrotropy also influences the reflection of EMWs from an optically active medium [1–15]. Alongside the signal described by Fresnel's formulae, a weak signal of polarization, perpendicular to the polarization of the incident wave, appears in the reflected wave. This signal gives information concerning the gyrotropy of the medium and it could be detected through appropriate differential methods [9, 15].

The present paper studies the problem of the reflection of EMWs from an optically active isotropic medium. The usual constitutive relations $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ and $\boldsymbol{B} = \mu \boldsymbol{H}$ are inadequate to describe the electromagnetic properties of isotropic chiral media. Several sets of constitutive equations have been proposed for use in conjunction with Maxwell's equations and respective boundary conditions in order to describe the interaction of light with an isotropic optically active medium [16].

Condon [17] proposed a symmetric set of constitutive relations:

$$\begin{aligned} \boldsymbol{D} &= \varepsilon \boldsymbol{E} - g\,\partial \boldsymbol{H}/\partial t \\ \boldsymbol{B} &= \mu \boldsymbol{H} + g\,\partial \boldsymbol{E}/\partial t \end{aligned} \tag{1}$$

g being the gyrotropic parameter, ε the dielectric permittivity and μ the magnetic

0953-8984/91/050529 + 10 $03.50

permeability of the medium. Silverman [4, 5] and Bassiri *et al* [6] used constitutive equations equivalent to (1) in conjunction with the standard Maxwell boundary conditions and obtained analytically exact self-consistent reflection and transmission amplitudes at the interface between an isotropic non-gyrotropic medium and an isotropic gyrotropic medium. We solve the same problem, as well as the boundary problem at the interface between an isotropic gyrotropic medium and an isotropic non-gyrotropic medium, and obtain explicit formulae for the reflection and refraction amplitudes in the realistic case of weak gyrotropy (section 2).

Another asymmetric set of constitutive equations was proposed by Born [18] and was used in [1, 2, 3, 7–11]:

$$\boldsymbol{D} = \varepsilon(\boldsymbol{E} + f\boldsymbol{\nabla} \times \boldsymbol{E}) \qquad \boldsymbol{B} = \mu\boldsymbol{H} \tag{2}$$

where f is the gyrotropic parameter of the medium.

This set of constitutive relations neglects the gyrotropic anisotropy of the magnetization, induced by the light, assuming the permeability μ to be a scalar (ordinarily $\mu = 1$). Such an assumption is justified by the invariance of the Maxwell equations at optical frequencies under transformations that intermix the polarization and magnetization terms, rescaling the gyrotropic parameter in the refractive indices of the RCP and LCP waves. Silverman [4, 5] has shown that the constitutive relations (2), when used with the standard Maxwell boundary conditions, fail to give a physically acceptable solution for the reflection and transmission of EMWs. A correct solution of the boundary problem may be obtained when relations (2) are used with modified boundary conditions [1, 2, 7].

In section 3 the possibility of using the reflection from a gyrotropic medium to study its gyrotropy is discussed. The applications of differential reflection methods are considered in the case of the usual reflection of LCP and RCP EMWs [9, 19, 20] (section 3.1) and in the case of total reflection (TR) [9] (section 3.2).

2. Reflection of EMWs from an optically active medium

2.1. Reflection at the non-active medium–optically active medium boundary

We shall consider the reflection of a plane EMW, propagating in a non-active isotropic medium, a, with a refractive index $n_0 = \sqrt{\varepsilon_0\mu_0}$ at the boundary with a gyrotropic isotropic medium, b, having the constitutive equations (1). Both media are considered to be non-absorbing.

The normal EMWs in the gyrotropic isotropic medium are RCP and LCP waves [5] of refractive indices

$$n^{\pm} = n \pm |\rho'|/2 \tag{3}$$

and polarization

$$E_y^{\pm} = \mp \mathrm{i}sE_x^{\pm} \tag{4}$$

where

$$n = \sqrt{\varepsilon\mu} \qquad \rho' = 2\omega g \qquad s = \rho'/|\rho'|. \tag{5}$$

The quantities $\varepsilon(\omega)$, $\mu(\omega)$ and $\rho'(\omega)$ have resonances for the frequencies of the dipole active excitations of the medium [9, 21].

Usually in reflection studies the intensity E of the electric field of the incident wave is represented as the sum of two polarizations: E_p (polarization parallel to the plane of incidence) and E_s (polarization perpendicular to the plane of incidence). In the non-active medium, a, a wave E'' with components E''_p and E''_s is reflected, the angle of reflection being equal to the angle θ of incidence (figure 1). In the gyrotropic medium, b, a RCP wave E^+ (E^+_p, $E^+_s = -\mathrm{i}sE^+_p$) and a LCP wave E^- (E^-_p, $E^-_s = \mathrm{i}sE^-_p$) are refracted at angles φ^+ and φ^-, respectively.

Snell's law is fulfilled because of the continuity conditions at the boundary:

$$n_0 \sin\theta = n^+ \sin\varphi^+ = n^- \sin\varphi^- = n \sin\varphi \tag{6}$$

where φ is the angle of refraction in the case where there is no gyrotropy of the medium b (at $\rho' = 0$, when $n^+ = n^- = n = \sqrt{\varepsilon\mu}$).

The boundary problem is solved by implementation of the standard boundary conditions of continuity of the tangential components of E and H across the surface:

$$E^a_t = E^b_t \qquad H^a_t = H^b_t. \tag{7}$$

The following formulae for the s and p components of the reflected wave E'' and for the amplitudes of the RCP and LCP refracted waves $E^\pm$ are obtained:

$$E''_s = AE_s + \mathrm{i}CE_p \tag{8a}$$

$$E''_p = BE_p - \mathrm{i}CE_s \tag{8b}$$

$$E^\pm_p = F^\pm E_p + \mathrm{i}G^\pm E_s. \tag{8c}$$

The generalized Fresnel coefficients A, B, C, $F^\pm$ and $G^\pm$ obtained, coincide with the results of Silverman [5] and are given in appendix 1.

The natural optical activity is a relatively weak effect and, for frequencies ω far from the resonances of the dipole active excitations of the medium, the inequality $|\rho'| \ll n$ holds (usually $\rho' \simeq 10^{-3}$–10^{-5}; see [21]). We shall restrict our considerations to the case of weak gyrotropy and we shall consider angles θ of incidence, which are not in the closest vicinities of the critical angle of TR (i.e. when $\cos\varphi \neq 0$). Then $\cos\varphi^\pm$ can be presented in the following way:

$$\cos\varphi^\pm = \cos\varphi \pm \delta \tag{9}$$

$$\delta = (|\rho'| \sin^2\varphi)/(2n\cos\varphi). \tag{10}$$

For the case of weak gyrotropy and non-magnetic media ($\mu = \mu_0 = 1$) the Fresnel amplitudes (see appendix 1, (A1)–(A5)) take the form

$$F^\pm = (N \mp sC)/2 \tag{11a}$$

$$G^\pm = (\pm sM - Cn_0/n)/2 \tag{11b}$$

$$A = (n_0\cos\theta - n\cos\varphi)/(n_0\cos\theta + n\cos\varphi) \tag{12a}$$

$$B = (n\cos\theta - n_0\cos\varphi)/(n\cos\theta + n_0\cos\varphi) \tag{12b}$$

$$M = (2n_0\cos\theta)/(n_0\cos\theta + n\cos\varphi) \tag{12c}$$

$$N = (2n_0\cos\theta)/(n\cos\theta + n_0\cos\varphi). \tag{12d}$$

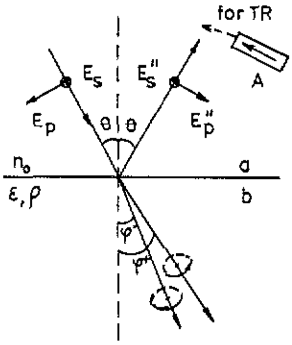

Figure 1. Reflection of an EMW at the boundary of an optically non-active isotropic medium a and an optically active isotropic medium b. E_p, E_s, E''_p and E''_s are the components of the electric field of the incident EMW and of the reflected EMW, φ^+ and φ^- are refraction angles of the normal EMW propagating in the gyrotropic medium b (+, RCP; −, LCP), n_0 is the refractive index of medium a, and ε and ρ' are the dielectric constant and gyrotropic parameter of the optically active medium b. The analyser A is introduced for TR measurements only; see section 3.2.

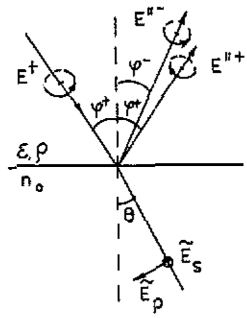

Figure 2. Reflection of an EMW at the boundary of an optically active medium and an optically non-active medium. A RCP EMW is reflected and two circularly polarized EMWs appear: E''^+ (of the same polarization) and E''^- (of the opposite polarization). $\varphi^- \neq \varphi^+$ (see equations (6) for the refraction law); $\tilde{E}_p$ and $\tilde{E}_s$ are the components of the refracted wave.

The coefficients A, B, M and N coincide with the well known Fresnel amplitudes for non-gyrotropic isotropic media. The coefficient

$$C = \rho'(n_0 \cos\theta \sin^2\varphi)/\cos\varphi\,(n_0\cos\theta + n\cos\varphi)(n_0\cos\varphi + n\cos\theta) \tag{13}$$

is a gyrotropic contribution proportional to the small parameter ρ' ($\simeq 10^{-3}$–10^{-5}).

The same results for the reflected and refracted EMWs are obtained in the case of weak gyrotropy, when Born's constitutive equations (2) (instead of Condon's equations (1)) are used in conjunction with modified boundary conditions, which take into account the gyrotropy of the medium b [1]:

$$E_t^a + E_t^b \qquad B_t^a = B_t^b - \mathrm{i}(\rho'/2c)\dot{E}_t^b. \tag{14}$$

So the optical activity of medium b introduces some new features in the reflection of EMWs. In the case when medium b is optically active, the independent transformation of the s and p polarizations is violated: the reflected s (or p) wave contains a weak contribution from the perpendicular p (or s) polarization of the incident wave. This contribution is described by the gyrotropic coefficient C (8a) and (8b).

In the case of a linearly polarized incident wave, e.g. an s wave ($E_s \neq 0$, $E_p = 0$) an elliptically polarized reflected wave appears, and because of the small values of the quantity $C \sim \rho'$ its ellipticity is strong. Measurements of the polarization and ellipsometric measurements could provide information about the gyrotropic parameter ρ' [11, 22]. In the case of a circularly polarized incident wave ($E_s = \pm \mathrm{i}E_p$) an elliptically polarized reflected wave appears and its components depend on the gyrotropic term C:

$$E''_s = \mathrm{i}(\pm A + C)E_p \qquad E''_p = (B \pm C)E_p.$$

2.2. *Reflection at the optically active isotropic medium–non-active medium boundary*

We shall consider the reflection and refraction of a RCP wave E^+ (E_p^+, $E_s^+ = -\mathrm{i}sE_p^+$), which is propagating in a gyrotropic isotropic medium, at the boundary with a non-active isotropic medium (figure 2). The gyrotropic medium is described by Condon's constitutive equations (1). In order to satisfy the standard Maxwell boundary conditions (7) two circularly polarized waves should be reflected:

(i) a RCP wave E''^{+} ($E_p''^{+}$, $E_s''^{+} = -\mathrm{i}sE_p''^{+}$) for which the angle of reflection is equal to the angle φ^{+} of incidence;

(ii) a LCP wave E''^{-} ($E_p''^{-}$, $E_s''^{-} = \mathrm{i}sE_p''^{-}$), for which the angle φ^{-} of reflection is slightly different from the angle φ^{+} of incidence (see equations (6)).

An elliptically polarized wave $\tilde{E}$ ($\tilde{E}_p$, $\tilde{E}_s$) is refracted at angle θ in the non-gyrotropic medium. The components of these waves are given by the formulae

$$E_p''^{+} = R^{+}E_p^{+} \tag{15a}$$

$$E_p''^{-} = P^{+}E_p^{+} \tag{15b}$$

$$\tilde{E}_s = M_1^{+}E_p^{+} \tag{15c}$$

$$\tilde{E}_p = N_1^{+}E_p^{+}. \tag{15d}$$

The expressions for the coefficients R^{+}, P^{+}, M_1^{+} and N_1^{+} are given in appendix 2. In the usual case of weak optical activity, at angles of incidence far from the critical angle (when (9) holds) for non-magnetic media ($\mu = \mu_0 = 1$) equations ((15*a*)–(15*d*)) are reduced to the following expressions:

$$E_p''^{+} = (R + r)E_p^{+} \tag{16a}$$

$$E_p''^{-} = (P + p)E_p^{+} \tag{16b}$$

$$\tilde{E}_s = -\mathrm{i}s(M_1 + sCn/n_0)E_p^{+} \tag{16c}$$

$$\tilde{E}_p = (N_1 + sC)E_p^{+} \tag{16d}$$

where

$$R = -\tfrac{1}{2}(A + B) \tag{17a}$$

$$P = \tfrac{1}{2}(A - B) \tag{17b}$$

$$M_1 = (2n\cos\varphi)/(n_0\cos\theta + n\cos\varphi) \tag{17c}$$

$$N_1 = (2n\cos\varphi)/(n\cos\theta + n_0\cos\varphi) \tag{17d}$$

$$r = \delta\cos\theta\,(n_0^2 + n^2)/(n_0\cos\theta + n\cos\varphi)(n_0\cos\varphi + n\cos\theta) \tag{17e}$$

$$p = \delta\cos\theta\,(n_0^2 - n^2)/(n_0\cos\theta + n\cos\varphi)(n_0\cos\varphi + n\cos\theta). \tag{17f}$$

All other notation in (16) and (17) is given in section 2.1.

Similarly, in the case of reflection of a LCP EMW E^{-} (E_p^{-}, $E_s^{-} = \mathrm{i}sE_p^{-}$), two reflected circularly polarized waves appear:

(i) a LCP wave E''^{-} ($E_p''^{-}$, $E_s''^{-} = \mathrm{i}sE_p''^{-}$) which is reflected at an angle equal to the angle φ^{-} of incidence;

(ii) a RCP wave E''^{+} ($E_p''^{+}$, $E_s''^{+} = -\mathrm{i}sE_p''^{+}$) with an angle φ^{+} of reflection not equal to φ^{-}.

The solution of the boundary problem gives the following results for the amplitudes of the reflected waves and for the components $\bar{E}_s$ and $\bar{E}_p$ of the wave which is refracted in medium a (see appendix 2):

$$E_p''^{+} = P^{-}E_p^{-} \tag{18a}$$

$$E_p''^{-} = R^{-}E_p^{-} \tag{18b}$$

$$\bar{E}_s = M_1^{-}E_p^{-} \tag{18c}$$

$$\bar{E}_p = N_1^{-}E_p^{-}. \tag{18d}$$

Assuming a weak optical activity and an angle of incidence far from the critical angle (when (9) holds) we transform equations (18*a*) and (18*d*) into the forms

$$E_p''^{+} = (P - p)E_p^{-} \tag{19a}$$

$$E_p''^{-} = (R - r)E_p^{-} \tag{19b}$$

$$\bar{E}_s = \mathrm{i}s(M_1 - sCn/n_0)E_p^{-} \tag{19c}$$

$$\bar{E}_p = (N_1 - sC)E_p^{-}. \tag{19d}$$

A comparison between equations (16) and (19) shows that the transformation of the incident circularly polarized wave into reflected and refracted waves is done in a non-identical way for the LCP and for the RCP normal EMWs.

3. Differential reflection methods

In the case of reflection of a linearly polarized wave, the gyrotropy excites a component of polarization perpendicular to the polarization of the incident EMW; see equations (8*a*) and (8*b*). The electric field $E''_\perp$ of this component is proportional to the gyrotropic parameter ρ', but its intensity $I'' \sim |E''_\perp|^2$ is proportional to the quantity ρ'^2. Because of the small values of this parameter ($\rho' \simeq 10^{-3}$–10^{-5}) the reflected component of perpendicular polarization contains a very small fraction of the intensity I_0 of the incident EMW:

$$I''_\perp \sim \rho'^2 I_0 \simeq (10^{-6}\text{–}10^{-10})I_0. \tag{20}$$

The weak intensity of the component of perpendicular polarization is an obstacle to its measurement and to the investigation of the optical activity through the reflection spectra.

Using the differential methods proposed below, a signal could be obtained which is linear in the quantity ρ' and is therefore considerably stronger.

3.1. Differential methods in the case of the usual reflection

We propose to measure a differential signal which is the difference between the intensities $I''(-)$ and $I''(+)$ of the waves reflected from the optically active medium, when the incident waves are circularly polarized (LCP and RCP) of equal intensities I_0. The components of these circularly polarized EMWs are connected through the relations

$$E_p^{\pm} = \pm\mathrm{i}E_s^{\pm} \qquad |E_s^{+}| = |E_s^{-}|. \tag{21}$$

The second equation expresses the equality between the intensities I_0 of the two circularly polarized waves. When equations (21) are substituted in equations (8*a*) and (8*b*), the following values of the components of the reflected EMW are obtained:

$$E_s''^{(\pm)} = E_s^{\pm}(A \mp C) \qquad E_p''^{(\pm)} = \mathrm{i}E_s^{\pm}(\pm B - C). \tag{22}$$

The total intensity I'' of the reflected EMW is determined with an accuracy up to terms which are linear in the quantity ρ' (i.e. linear in C). In this way we find that

$$I''^{(\pm)} = I_s''^{(\pm)} + I_p''^{(\pm)} = \tfrac{1}{2}I_0[(A^2 \mp 2AC) + (B^2 \mp 2BC)]. \tag{23}$$

For the difference between the two signals obtained from the reflection of a LCP $(-)$ and a RCP $(+)$ EMW of equal intensities, the following expression is found:

$$I''(-) - I''(+) = 2I_0C(A + B). \tag{24}$$

The differential signal (24) is proportional to the gyrotropic parameter ρ' of the isotropic optically active medium (see (13)). This makes it possible to determine the values of ρ' and its frequency dependence. The dependence of (24) on the angle θ of incidence is complex and represents a combination of the respective dependences in Fresnel's formulae for the reflection for the s and p components.

In the proposed differential method the difference in the reflection properties of the gyrotropic medium for EMWs of different circular polarizations is investigated. A similar non-identical reaction of the optically active medium is manifest also in other phenomena related to gyrotropy: the circular birefringence, the circular dichroism (difference in the absorption), the Raman optical activity (difference in the Raman scattering), etc. The different reflections of LCP and RCP EMWs from a gyrotropic medium may be called the reflection optical activity [3, 9]. It is the quantity proportional to the signal (24) which was measured in [3]. Silverman [19, 20] proposed an experimental method based on phase modulation and synchronous detection to measure the differential reflection of incident LCP and RCP waves and other signals, demonstrating the asymmetric response of the gyrotropic medium to orthogonal components of linearly polarized light (incident TE and TM light). The experimental results for differential reflectances from a gyrotropic medium measured by the use of a photoelastic (or birefringent) modulator have been given in [20].

3.2. Differential methods in the case of TR ($n_0 > n$)

The TR (at $\sin\theta > n/n_0$) is treated by considering in equations (12) and (13) the factor $\cos\varphi$ to be imaginary:

$$\cos\varphi = \mathrm{i}\alpha = \mathrm{i}\sqrt{(n_0^2\sin^2\theta)/n^2 - 1} \tag{25}$$

(see the analogous treatment of TR in [18]). Again we exclude from our considerations the closest vicinities of the critical angle of TR, i.e. we treat the quantity δ in equation (10) as a small gyrotropic addition. The intensities I_s'' and I_p'' of the s and p components of the reflected wave are obtained by calculating the quantities $|E_s''|^2$ and $|E_p''|^2$ from equations (8*a*) and (8*b*) when relation (25) holds. We find that the gyrotropic non-absorbing medium does not attenuate the TR, but it acts like a transformer (of low efficiency) of the incident p-polarized (s-polarized) wave into a reflected s-polarized (p-polarized) wave:

$$I_s'' = I_s^0 + \eta I_p'' = I_p^0 - \eta \tag{26}$$

where I_s^0 and I_p^0 are the intensities of the s and p components of the incident EMW and η is an addition, proportional to the gyrotropic parameter ρ'.

In order to obtain a signal giving information about the gyrotropy in the case of TR, it is necessary to use an additional analyser (see figure 1) which absorbs the s or p component of the reflected beam. (It is obvious that without an analyser the total intensity of the reflected wave is equal to the intensity of the incident wave: $I'' = I''_s + I''_p = I^0_s + I^0_p = I_0$.)

We propose two modifications of the differential spectroscopy in the case of TR.

3.2.1. TR of circularly polarized waves. The difference in the intensities of the reflected s waves (or p waves) is measured when LCP(−) and RCP(+) waves of equal intensities I_0 are incident upon the gyrotropic medium. The method makes use of the property of the gyrotropic medium to transform differently—in the case of TR—the LCP and RCP waves into elliptically polarized waves of very small ellipticity. The calculations of the differential signal are similar to the calculations in section 3.1. Here we note again that in the case of TR the difference in the intensities only of the s or only of the p components of the reflected signal should be measured (but not of the total reflected signals, as is done in section 3.1). Thus we obtain

$$I''_s(+) - I''_s(-) = 2\eta = (I_0/2)\rho'[n_0^3 \sin^2(2\theta)]/(n_0^2 - n^2)(n_0^2 \sin^2\theta - n^2\cos^2\theta). \tag{27}$$

The differential signal provides information about the gyrotropic parameter ρ'. The quantity (27) decreases monotonically from the angle θ of incidence equal to $\theta_{crit} = \sin^{-1}(n/n_0)$ to zero at $\theta = \pi/2$.

3.2.2. TR of linearly polarized waves. The difference in the intensities of the s (or p) components of the reflected waves is measured when linearly polarized incident waves of azimuths γ and $-\gamma(E_s = E\sin\gamma,\ E_p = E\cos\gamma)$ are reflected. The quantity η in (26) is proportional to $\sin(2\gamma)$ and the following non-zero differential signal appears (it can be calculated in the same way as in section 3.2.1):

$$I''_s(\gamma) - I''_s(-\gamma) = 2\eta = I_0 2\rho' \sin(2\gamma)(n_0^4 \sin^4\theta\cos\theta)/\sqrt{n_0^2\sin^2\theta - n^2} \times (n_0^2 - n^2)(n_0^2\sin^2\theta - n^2\cos^2\theta). \tag{28}$$

The differential signal (28) reaches its maximum when the azimuth of the linearly polarized wave is $\gamma = \pm\pi/4$. This signal may increase when the angle of incidence is close to the critical angle: $\theta \approx \theta_{crit} = \sin^{-1}(n/n_0)$, $\theta > \theta_{crit}$. The quantity (28) increases when the ratio $n/n_0 > 1$ is close to unity.

The proposed method—TR of linearly polarized waves—is used in [15] to investigate the optical activity of a liquid-crystal substance near the frequencies of the intramolecular vibrations. In this case the gyrotropic medium is absorbing and the TR is attenuated. This situation requires special consideration.

4. Conclusion

The present paper treats the influence of the optical activity upon the reflection of EMWs at the boundary between an isotropic non-active medium and an isotropic optically active medium (a solution or a cubic crystal). As a result of the gyrotropy the electric field of the reflected wave contains a weak contribution of polarization perpendicular to the polarization of the incident wave. Thus in the case of reflection of LCP and RCP waves the non-identical reaction of the gyrotropic medium will occur—reflection optical activity. This phenomenon is analogous to circular birefringence and circular dichroism when an EMW propagates through an optically active medium.

At the boundary between the gyrotropic isotropic medium and the non-active medium, two reflected LCP and RCP waves appear when a LCP (or RCP) normal EMW is reflected. The theoretical investigation of the reflection of EMWs at a non-active medium–optically active medium boundary and an optically active medium–non-active medium boundary served as a basis for the study of the multibeam interference of EMWs from an optically active Fabry–Pérot etalon [23].

The possibility of investigating the gyrotropic properties by means of appropriate differential reflection methods is pointed out in this paper. The signal obtained by the differential methods is linear in the gyration parameter $\rho'(\omega)$ and is within the sensitivity range of modern measuring devices. As far as the experimental test of the obtained formulae is concerned, the differential reflectance from isotropic optically active samples is measured in [3] and [20]. Interesting physical results are obtained by means of differential methods for uniaxial media in [15] (see section 3.2.2) as well as in [12, 13] (for optical activity in the exciton region of CdS).

Non-oriented solutions and cubic crystals do not exhibit linear birefringence, but they are also isotropic with respect to their optical activity [21] (every direction in them serves as the optical axis). The investigation of reflections from isotropic gyrotropic media is facilitated by the absence of a fixed direction of the optical axis, which eliminates the necessity to search for an appropriate cut of the crystal and to orient the sample. However, for optically active non-oriented solutions, additional difficulties caused by the low reflectivity of liquid surfaces may arise (see [3], where the realized modification of the differential reflection method proved successful for crystal samples but failed for liquid samples). At the same time the transition region between the solution and its vapours may not be sharp and then a more complex theory should be developed to take into account this influence. A model in which the effect of the transition layer is considered may be found in [1].

The study of the peculiarities of the reflection from optically active media allows the introduction of a new method for the investigation of gyrotropy—reflection optical activity. Knowledge of these peculiarities is the basis for the correct interpretation of the reflection spectra of media possessing a natural optical activity or gyrotropy induced by external fields or by technological factors.

Appendix 1

$$A = [\cos^2\theta - \cos\varphi^+ \cos\varphi^- - \tfrac{1}{2}\cos\theta\,(\cos\varphi^+ + \cos\varphi^-)(q - q^{-1})]/d \tag{A1}$$

$$B = [\cos^2\theta - \cos\varphi^+ \cos\varphi^- + \tfrac{1}{2}\cos\theta\,(\cos\varphi^+ + \cos\varphi^-)(q - q^{-1})]/d \tag{A2}$$

$$C = \cos\theta\,(\cos\varphi^+ - \cos\varphi^-)/d \tag{A3}$$

$$F^{\pm} = \cos\theta\,[\cos\varphi^{\mp} + (\cos\theta)/q]/d \tag{A4}$$

$$G^{\pm} = \pm\cos\theta\,[\cos\theta + (\cos\varphi^{\mp})/q]/d \tag{A5}$$

where

$$q = n\mu_0/n_0\mu \tag{A6}$$

$$d = \cos^2\theta + \cos\varphi^+ \cos\varphi^- + \tfrac{1}{2}\cos\theta\,(\cos\varphi^+ + \cos\varphi^-)(q + q^{-1}) \tag{A7}$$

All other notation is given in the text.

Appendix 2

$$R^\pm = [\cos\varphi^+ \cos\varphi^- - \cos^2\theta + \tfrac{1}{2}\cos\theta\,(\cos\varphi^\pm - \cos\varphi^\mp)(q + q^{-1})]/d \tag{A8}$$

$$P^\pm = \cos\theta\cos\varphi^\pm\,(q - q^{-1})/d \tag{A9}$$

$$M_\uparrow^\pm = \mp \mathrm{i}2s\cos\varphi^\pm\,(\cos\varphi^\mp + q\cos\theta)/d \tag{A10}$$

$$N_\uparrow^\pm = 2\cos\varphi^\pm\,(\cos\theta + q\cos\varphi^\mp)/d \tag{A11}$$

where q and d are given in appendix 1 ((A6) and (A7)) and the other notation is given in the text.

References

[1] Agranovich V M and Ydson V I 1972 *Opt. Commun.* **5** 422–4
[2] Bokut B V and Serdukov A N 1971 *Zh. Eksp. Teor. Fiz.* **61** 1808–13
[3] Einhorn A J, Phelps F W and Pao Y H 1973 *Chem. Phys.* **1** 277–96
[4] Silverman M P 1985 *Lett. Nuovo Cimento* **43** 378–82
[5] Silverman M P 1986 *J. Opt. Soc. Am.* A **3** 830–7
[6] Bassiri S, Papas C H and Engheta N 1988 *J. Opt. Soc. Am.* A **5** 1450–9
[7] Bishop M F and Maradudin A A 1977 *Solid State Commun.* **23** 507–10
[8] Takizawa T 1981 *J. Phys. Soc. Japan* **50** 3054–62
[9] Lalov I J and Miteva A I 1986 *J. Chem. Phys.* **85** 5505–11
[10] Lalov I J and Miteva A I 1988 *Proc. 10th Natl. Conf. on Molecular Spectroscopy (Blagoevgrad)* (Singapore: World Scientific) pp 307–38
[11] Lalov I J 1985 *Phys. Rev.* B **32** 6884–90
[12] Ivchenko E L, Permogorov S A and Selkin A V 1978 *Solid State Commun.* **28** 345–8
[13] Ivchenko E L 1982 *Excitons* ed E I Rashba and M D Sturge (Amsterdam: North-Holland) ch 4, pp 141–79
[14] Mashliatina T M, Nedzvetzkii D S and Selkin A V 1978 *Zh. Eksp. Teor. Fiz. Pis. Red.* **27** 573–5
[15] Jordanov B and Schrader B 1986 *J. Mol. Struct.* **141** 297–300
[16] Lakhtakia A, Varadan V V and Varadan V K 1988 *J. Opt. Soc. Am.* A **5** 175–84
[17] Condon E U 1937 *Rev. Mod. Phys.* **9** 432–57
[18] Born M 1972 *Optik* (Berlin: Springer) p 412
[19] Silverman M P and Black T C 1987 *Phys. Lett.* **126A** 171–6
[20] Silverman M P, Ritchie N, Cushman G M and Fisher B 1988 *J. Opt. Soc. Am.* A **5** 1852–62
[21] Agranovich V M and Ginzburg 1984 *Crystal Optics with Spatial Dispersion and Excitons* (Berlin: Springer) ch 4
[22] Pevtsov A B and Selkin A V 1983 *Fiz. Tvod. Tela* **25** 157–62
[23] Lalov I J and Miteva A I 1990 *J. Mod. Opt.* at press